DIKTAT TRAKSI LISTRIK

CHAPTER 43

ELECTRIC TRACTION

Learning Objectives

- 1 ➢ General
- 2 ➢ Traction System
- 3 ➢ Direct Steam Engine Drive
- 4 ➢ Advantages of Electric Traction
- 5 ➢ Saving in High Grade Coal
- 6 ➢ Disadvantages of Electric Traction
- 7 ➢ System of Railway Electrification
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In electric traction, driving force (or tractive force) is generated by electricity, using electric motors. Electric trains, trams, trolley, buses and battery run cars are some examples where electric traction is employed.

43.1. General

By electric traction is meant locomotion in which the driving (or tractive) force is obtained from electric motors. It is used in electric trains, tramcars, trolley buses and diesel-electric vehicles etc. Electric traction has many advantages as compared to other non-electrical systems of traction including steam traction.

43.2. Traction Systems

Broadly speaking, all traction systems may be classified into two categories :

(*a*) **non-electric traction systems**

They do not involve the use of electrical energy at ***any stage***. Examples are : steam engine drive used in railways and internal-combustion-engine drive used for road transport.

The above picture shows a diesel train engine. These engines are now being rapidly replaced by electric engines .

(*b*) **electric traction systems**

They involve the use of electric energy at some stage or the other. They may be further subdivided into two groups :

1. First group consists of self-contained vehicles or locomotives. Examples are : battery-electric drive and diesel-electric drive etc.
2. Second group consists of vehicles which receive electric power from a distribution network fed at suitable points from either central power stations or suitably-spaced sub-stations. Examples are : railway electric locomotive fed from overhead ac supply and tramways and trolly buses supplied with dc supply.

43.3. Direct Steam Engine Drive

Though losing ground gradually due to various reasons, steam locomotive is still the most widely-adopted means of propulsion for railway work. Invariably, the reciprocating engine is employed because

1. it is inherently simple.
2. connection between its cylinders and the driving wheels is simple.
3. its speed can be controlled very easily.

tion has the following advantages :

1. **Cleanliness.** Since it does not produce any smoke or corrosive fumes, electric traction is most suited for underground and tube railways. Also, it causes no damage to the buildings and other apparatus due to the absence of smoke and flue gases.

2. **Maintenance Cost.** The maintenance cost of an electric locomotive is nearly 50% of that for a steam locomotive. Moreover, the maintenance time is also much less.

3. **Starting Time.** An electric locomotive can be started at a moment's notice whereas a steam locomotive requires about two hours to heat up.

4. **High Starting Torque.** The motors used in electric traction have a very high starting torque. Hence, it is possible to achieve higher accelerations of 1.5 to 2.5 km/h/s as against 0.6 to 0.8 km/h/s in steam traction. As a result, we are able to get the following additional advantages:

(*i*) high schedule speed

(*ii*) increased traffic handling capacity

(*iii*) because of (*i*) and (*ii*) above, less terminal space is required—a factor of great importance in urban areas.

5. **Braking.** It is possible to use regenerative braking in electric traction system. It leads to the following advantages :

(*i*) about 80% of the energy taken from the supply during ascent is returned to it during descent.

(*ii*) goods traffic on gradients becomes safer and speedier.

(*iii*) since mechanical brakes are used to a very small extent, maintenance of brake shoes, wheels, tyres and track rails is considerably reduced because of less wear and tear.

6. **Saving in High Grade Coal.** Steam locomotives use costly high-grade coal which is not so abundant. But electric locomotives can be fed either from hydroelectric stations or pit-head thermal power stations which use cheap low-grade coal. In this way, high-grade coal can be saved for metallurgical purposes.

7. **Lower Centre of Gravity.** Since height of an electric locomotive is much less than that of a steam locomotive, its centre of gravity is comparatively low. This fact enables an electric locomotive to negotiate curves at higher speeds quite safely.

8. **Absence of Unbalanced Forces.** Electric traction has higher coefficient of adhesion since there are no unbalanced forces produced by reciprocating masses as is the case in steam traction. It not only reduces the weight/kW ratio of an electric locomotive but also improves its riding quality in addition to reducing the wear and tear of the track rails.

43.7. Disadvantages of Electric Traction

1. The most vital factor against electric traction is the initial high cost of laying out overhead electric supply system. Unless the traffic to be handled is heavy, electric traction becomes uneconomical.

2. Power failure for few minutes can cause traffic dislocation for hours.

3. Communication lines which usually run parallel to the power supply lines suffer from electrical interference. Hence, these communication lines have either to be removed away from the rail track or else underground cables have to be used for the purpose which makes the entire system still more expensive.

4. Electric traction can be used only on those routes which have been electrified. Obviously, this restriction does not apply to steam traction.

5. Provision of a negative booster is essential in the case of electric traction. By avoiding the

flow of return currents through earth, it curtails corrosion of underground pipe work and interference with telegraph and telephone circuits.

43.8. Systems of Railway Electrification

Presently, following four types of track electrification systems are available :

1. Direct current system—600 V, 750 V, 1500 V, 3000 V
2. Single-phase ac system—15-25 kV, $16\frac{2}{3}$, 25 and 50 Hz
3. Three-phase ac system—3000-3500 V at $16\frac{2}{3}$ Hz
4. Composite system—involving conversion of single-phase ac into 3-phase ac or dc.

43.9. Direct Current System

Direct current at 600-750 V is universally employed for tramways in urban areas and for many suburban railways while 1500-3000 V dc is used for main line railways. The current collection is from third rail (or conductor rail) up to 750 V, where large currents are involved and from overhead wire for 1500 V and 3000 V, where small currents are involved. Since in majority of cases, track (or running) rails are used as the return conductor, only one conductor rail is required. Both of these contact systems are fed from substations which are spaced 3 to 5 km for heavy suburban traffic and 40-50 km for main lines operating at higher voltages of 1500 V to 3000 V. These sub-stations themselves receive power from 110/132 kV, 3-phase network (or grid). At these substations, this high-voltage 3-phase supply is converted into low-voltage 1-phase supply with the help of Scott-connected or V-connected 3-phase transformers (Art. 31.9). Next, this low ac voltage is converted into the required dc voltage by using suitable rectifiers or converters (like rotary converter, mercury-arc, metal or semiconductor rectifiers). These substations are usually automatic and are remote-controlled.

The dc supply so obtained is fed via suitable contact system to the traction motors which are either dc series motors for electric locomotive or compound motors for tramway and trolley buses where regenerative braking is desired.

It may be noted that for ***heavy suburban service***, low voltage dc system is undoubtedly superior to 1-phase ac system due to the following reasons :

1. dc motors are better suited for frequent and rapid acceleration of heavy trains than ac motors.
2. dc train equipment is lighter, less costly and more efficient than similar ac equipment.
3. when operating under similar service conditions, dc train consumes less energy than a 1-phase ac train.
4. the conductor rail for dc distribution system is less costly, both initially and in maintenance than the high-voltage overhead ac distribution system.
5. dc system causes no electrical interference with overhead communication lines.

The only disadvantage of dc system is the necessity of locating ac/dc conversion sub-stations at relatively short distances apart.

43.10. Single-Phase Low-frequency AC System

In this system, ac voltages from 11 to 15 kV at $16\frac{2}{3}$ or 25 Hz are used. If supply is from a generating station exclusively meant for the traction system, there is no difficulty in getting the electric supply of $16\frac{2}{3}$ or 25 Hz. If, however, electric supply is taken from the high voltage transmission lines at 50 Hz, then in addition to step-down transformer, the substation is provided with a frequency converter. The frequency converter equipment consists of a 3-phase synchronous motor which drives a 1-phase alternator having or 25 Hz frequency.

The 15 kV $16\frac{2}{3}$ or 25 Hz supply is fed to the electric locomotor via a single over-head wire (running rail providing the return path).

A step-down transformer carried by the locomotive reduces the 15-kV voltage to 300-400 V for feeding the ac series motors. Speed regulation of ac series motors is achieved by applying variable voltage from the tapped secondary of the above transformer.

Low-frequency ac supply is used because apart from improving the commutation properties of ac motors, it increases their efficiency and power factor. Moreover, at low frequency, line reactance is less so that line impedance drop and hence line voltage drop is reduced. Because of this reduced line drop, it is feasible to space the substations 50 to 80 km apart. Another advantage of employing low frequency is that it reduces telephonic interference.

41.11. Three-phase Low-frequency AC System

It uses 3-phase induction motors which work on a 3.3 kV, $16\frac{2}{3}$ Hz supply. Sub-stations receive power at a very high voltage from 3-phase transmission lines at the usual industrial frequency of 50 Hz. This high voltage is stepped down to 3.3 kV by transformers whereas frequency is reduced from 50 Hz to $16\frac{2}{3}$ Hz by frequency converters installed at the sub-stations. Obviously, this system employs *two* overhead contact wires, the track rail forming the third phase (of course, this leads to insulation difficulties at the junctions).

Induction motors used in the system are quite simple and robust and give trouble-free operation. They possess the mérits of high efficiency and of operating as a generator when driven at speeds above the synchronous speed. Hence, they have the property of automatic regenerative braking during the descent on gradients. However, it may be noted that despite all its advantages, this system has not found much favour and has, in fact, become obsolete because of its certain inherent limitations given below :

1. the overhead contact wire system becomes complicated at crossings and junctions.
2. constant-speed characteristics of induction motors are not suitable for traction work.
3. induction motors have speed/torque characteristics similar to dc shunt motors. Hence, they are not suitable for parallel operation because, even with little difference in rotational speeds caused by unequal diameters of the wheels, motors will becomes loaded very unevenly.

43.12. Composite System

Such a system incorporates good points of two systems while ignoring their bad points. Two such composite systems presently in use are :

1. 1-phase to 3-phase system also called Kando system
2. 1-phase to dc system.

43.13. Kando System

In this system, single-phase 16-kV, 50 Hz supply from the sub-station is picked up by the locomotive through the single overhead contact wire. It is then converted into 3-phase ac supply at the same frequency by means of phase converter equipment carried on the locomotives. This 3-phase supply is then fed to the 3-phase induction motors.

As seen, the complicated overhead two contact wire arrangement of ordinary 3-phase system is replaced by a single wire system. By using silicon controlled rectifier as inverter, it is possible to get variable-frequency 3-phase supply at 1/2 to 9 Hz frequency. At this low frequency, 3-phase motors develop high starting torque without taking excessive current. In view of the above, Kando system is likely to be developed further.

43.14. Single-phase AC to DC System

This system combines the advantages of high-voltage ac distribution at industrial frequency with the dc series motors traction. It employs overhead 25-kV, 50-Hz supply which is stepped down by the transformer installed in the locomotive itself. The low-voltage ac supply is then converted into dc supply by the rectifier which is also carried on the locomotive. This dc supply is finally fed to dc series traction motor fitted between the wheels. The system of traction employing 25-kV, 50-Hz, 1-phase ac supply has been adopted for all future track electrification in India.

43.15. Advantages of 25-kV, 50-Hz AC System

Advantages of this system of track electrification over other systems particularly the dc system are as under :

1. Light Overhead Catenary

Since voltage is high (25 kV), line current for a given traction demand is less. Hence, cross-section of the overhead conductors is reduced. Since these small-sized conductors are light, supporting structures and foundations are also light and simple. Of course, high voltage needs higher insulation which increases the cost of overhead equipment (OHE) but the reduction in the size of conductors has an overriding effect.

2. Less Number of Substations

Since in the 25-kV system, line current is less, line voltage drop which is mainly due to the resistance of the line is correspondingly less. It improves the voltage regulation of the line which fact makes larger spacing of 50-80 km between sub-stations possible as against 5-15 km with 1500 V dc system and 15-30 km with 3000 V dc sysem. Since the required number of substations along the track is considerably reduced, it leads to substantial saving in the capital expenditure on track electrification.

3. Flexibility in the Location of Substations

Larger spacing of substations leads to greater flexibility in the selection of site for their proper location. These substations can be located near the national high-voltage grid which, in our country, fortunately runs close to the main railway routes. The substations are fed from this grid thereby saving the railway administration lot of expenditure for erecting special transmission lines for their substations. On the other hand, in view of closer spacing of dc substations and their far away location, railway administration has to erect its own transmission lines for taking feed from the national grid to the substations which consequently increases the initial cost of electrification.

4. Simplicity of Substation Design

In ac systems, the substations are simple in design and layout because they do not have to install and maintain rotary converters or rectifiers as in dc systems. They only consist of static transformers alongwith their associated switchgear and take their power directly from the high-voltage national grid running over the length and breadth of our country. Since such sub-stations are remotely controlled, they have few attending personnel or even may be unattended.

5. **Lower Cost of Fixed Installations**

The cost of fixed installations is much less for 25 kV ac system as compared to dc system. In fact, cost is in ascending order for 25 kV ac, 3000 V dc and 1500 V dc systems. Consequently, traffic densities for which these systems are economical are also in the ascending order.

6. **Higher Coefficient of Adhesion**

The straight dc locomotive has a coefficient of adhesion of about 27% whereas its value for ac rectifier locomotive is nearly 45%. For this reason, a lighter ac locomotive can haul the same load as a heavier straight dc locomotive. Consequently, ac locomotives are capable of achieving higher speeds in coping with heavier traffic.

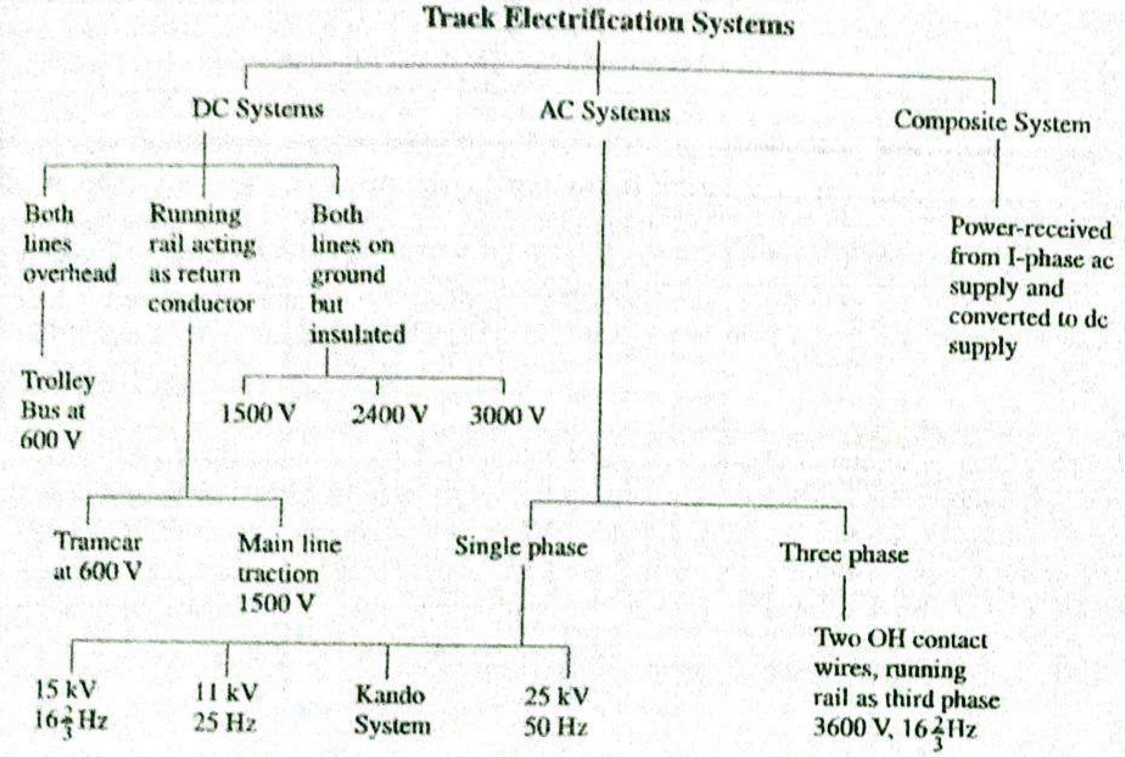
7. **Higher Starting Efficiency**

An ac locomotive has higher starting efficiency than a straight dc locomotive. In dc locomotive supply voltage at starting is reduced by means of ohmic resistors but by on-load primary or secondary tap-changer in ac locomotives.

43.16. Disadvantages of 25-kV AC System

1. Single-phase ac system produces both current and voltage unbalancing effect on the supply.
2. It produces interference in telecommunication circuits. Fortunately, it is possible at least to minimize both these undesirable effects.

Different track electrification systems are summarised below :



43.17. Block Diagram of an AC Locomotive

The various components of an ac locomotive running on single-phase 25-kV, 50-Hz ac supply are numbered in Fig. 43.1.

1. OH contact wire

2. pantograph
3. circuit breakers
4. on-load tap-changers
5. transformer
6. rectifier
7. smoothing choke
8. dc traction motors.

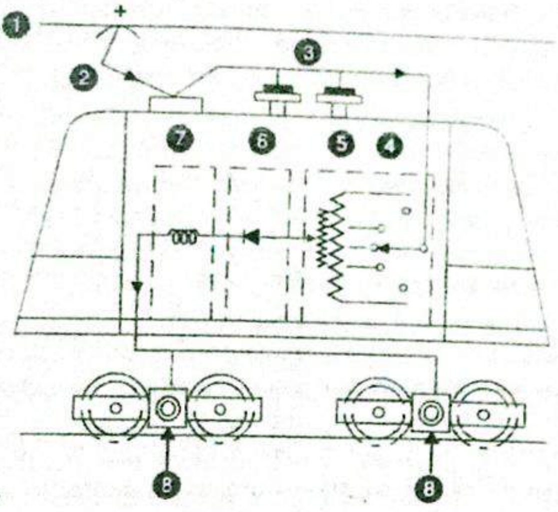


Fig. 43.1

As seen, power at 25 kV is taken via a pantograph from the overhead contact wire and fed to the step-down transformer in the locomotive. The low ac voltage so obtained is converted into pulsating dc voltage by means of the rectifier. The pulsations in the dc voltage are then removed by the smoothing choke before it is fed to dc series traction motors which are mounted between the wheels.

The function of circuit breakers is to immediately disconnect the locomotive from the overhead supply in case of any fault in its electrical system. The on-load tap-changer is used to change the voltage across the motors and hence regulate their speed.

43.18. The Tramways

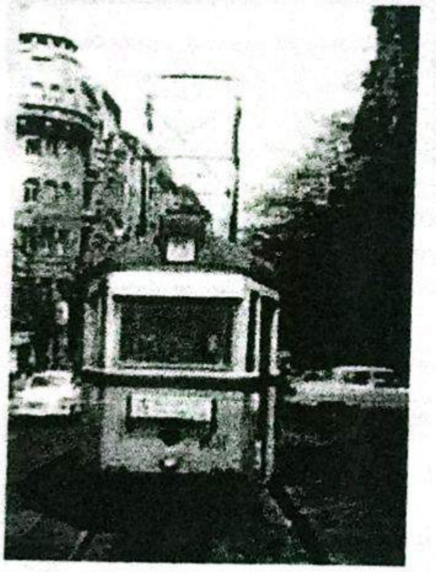
It is the most economical means of transport for very dense traffic in the congested streets of large cities. It receives power through a bow collector or a grooved wheel from an overhead conductor at about 600 V dc, the running rail forming the return conductor. It is provided with at least two driving axles in order to (*i*) secure necessary adhesion (*ii*) start it from either end and (*iii*) use two motors with series-parallel control. Two drum-type controllers, one at each end, are used for controlling the tramcar. Though these controllers are connected in parallel, they have suitable interlocking arrangement meant to prevent their being used simultaneously.

Tramcars are being replaced by trolley-buses and internal-combustion-engined omnibuses because of the following reasons :

1. tramcars lack flexibility of operation in congested areas.
2. the track constitutes a source of danger to other road users.

43.19. The Trolleybus

It is an electrically-operated pneumatic-tyred vehicle which needs ***no track in the roadway***. It receives its power at 600 V dc from two overhead contact wires. Since adhesion between a rubber-tyred wheel and ground is sufficiently high, only a single driving axle and, hence, a single motor is used. The trolleybus can manoeuvre through traffic a metre or two on each side of the centre line of the trolley wires.



Trolley Bus

43.20. Overhead Equipment (OHE)

Broadly speaking, there are two systems of current collection by a traction unit :

(*i*) third rail system and (*ii*) overhead wire system.

It has been found that current collection from overhead wire is far superior to that from the third rail. Moreover, insulation of third rail at high voltage becomes an impracticable proposition and endangers the safety of the working personnel.

The simplest type of OHE consists of a single contact wire of hard drawn copper or silico-bronze supported either by bracket or an overhead span. To facilitate connection to the supports, the wire is grooved as shown in Fig. 43.2. Because there is appreciable sag of the wire between supports, it limits the speed of the traction unit to about 30 km/h. Hence, single contact wire system is suitable for tramways and in complicated yards and terminal stations where speeds are low and simplicity of layout is desirable.

For collection of current by high-speed trains, the contact (or trolley) wire has to be kept level without any abrupt changes in its height between the supporting structures. It can be done by using the single catenary system which consists of one catenary or messenger wire of steel with high sag and the trolley (or contact) wire supported from messenger wire by means of droppers clipped to both wires as shown in Fig. 43.3.

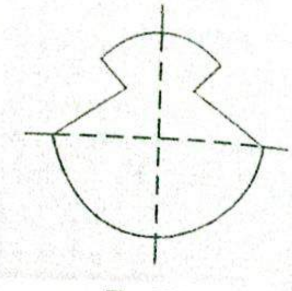


Fig. 43.2

43.21. Collector Gear for OHE

The most essential requirement of a collector is that it should keep continuous contact with trolley wire at all speeds. Three types of gear are in common use :

1. trolley collector 2. bow collector and 3. pantograph collector.

To ensure even pressure on OHE, the gear equipment must, be flexible in order to follow variations in the sag of the contact wire. Also, reasonable precautions must be taken to prevent the collector from leaving the overhead wire at points and crossings.

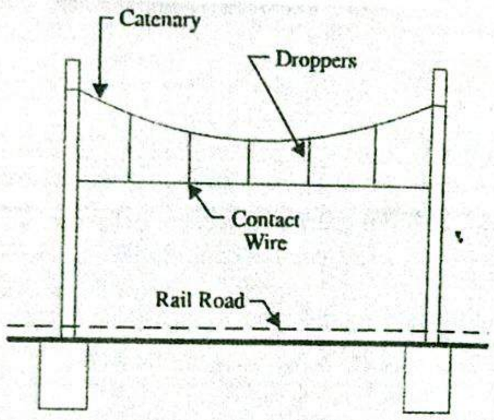


Fig. 43.3

43.22. The Trolley Collector

This collector is employed on tramways and trolley buses and is mounted on the roof of the vehicle. Contact with the OH wire is made by means of either a grooved wheel or a sliding shoe carried at the end of a light trolley pole attached to the top of the vehicle and held in contact with OH wire by means of a spring. The pole is hinged to a swivelling base so that it may be reversed for reverse running thereby making it unnecessary for the trolley wire to be accurately maintained above the centre of the track. Trolley collectors always operate in the trailing position.

The trolley collector is suitable for low speeds upto 32 km/h beyond which there is a risk of its jumping off the OH contact wire particularly at points and crossing.

43.23. The Bow Collector

It can be used for higher speeds. As shown in Fig. 43.4, it consists of two roof-mounted trolley poles at the ends of which is placed a light metal strip (or bow) about one metre long for current collection. The collection strip is purposely made of soft material (copper, aluminium or carbon) in order that most of the wear may occur on it rather than on the trolley wire. The bow collector also operates in the trailing position. Hence, it requires provision of either duplicate bows or an arrangement for reversing the bow for running in the reverse direction. Bow collector is not suitable for railway work where speeds up to 120 km/h and currents up to 3000 A are encountered. It is so because the inertia of the bow collector is too high to ensure satisfactory current collection.

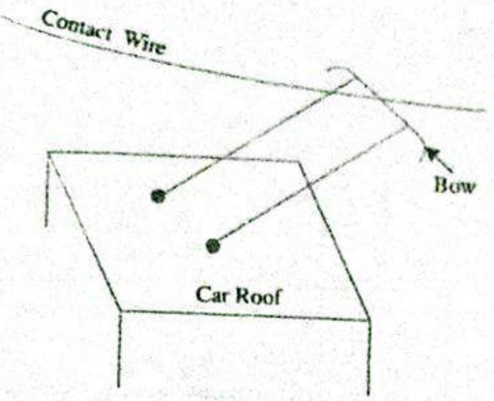


Fig. 43.4

43.24. The Pantograph Collector

Its function is to maintain link between overhead contact wire and power circuit of the electric locomotive at different speeds under all wind conditions and stiffness of OHE. It means that positive pressure has to be maintained at all times to avoid loss of contact and sparking but the pressure must be as low as possible in order to minimize wear of OH contact wire.

A 'diamond' type single-pan pantograph is shown in Fig. 43.5. It consists of a pentagonal framework of high-tensile alloy-steel tubing. The contact portion consists of a pressed steel pan fitted with renewable copper wearing strips which are forced against the OH contact wire by the upward action of pantograph springs. The pantograph can be raised or lowered from cabin by air cylinders.

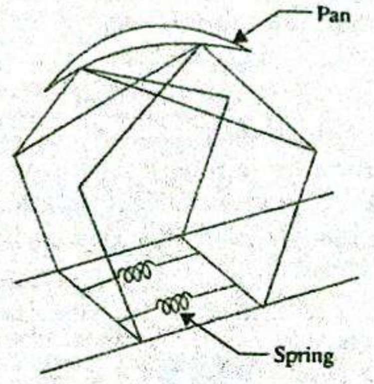


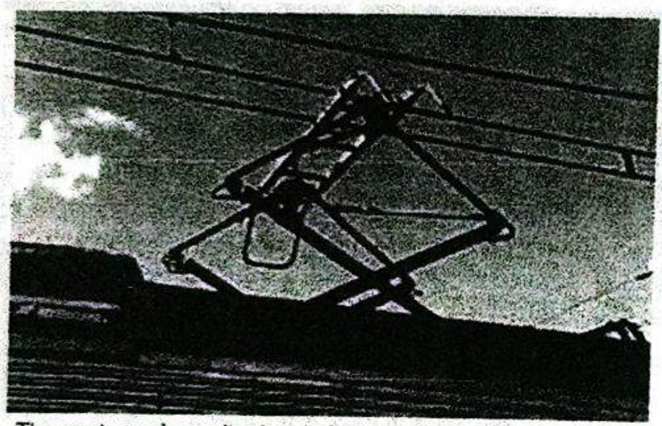
Fig. 43.5

43.25. Conductor Rail Equipment

The conductor rails may be divided into three classes depending on the position of the contact surface which may be located at the top, bottom or side of the rail. The top contact rail is adopted universally for 600 V dc electrification. The side contact rail is used for 1200 V dc supply. The under contact rail has the advantage of being protected from snow, sleet and ice.

Fig. 43.6 shows the case when electric supply is collected from the top of an insulated conductor rail *C* (of special high-conductivity steel) running parallel to the track at a distance of 0.3 to 0.4 m from the running rail (*R*) which forms the return path. *L* is the insulator and *W* is the wooden protection used at stations and crossings.

The current is collected from top surface of the rail by flat steel shoes (200 mm × 75 mm), the necessary contact pressure being obtained by gravity. Since it is not always possible to provide conductor rail on the same side of the track, shoes are provided on both sides of the locomotive or train. Moreover two shoes are provided on each side in order to avoid current interruption at points and crossings where there are gaps in the running rail.



The pantograph mechanism helps to maintain a link between the overhead contact wire and power circuit of the electric locomotive

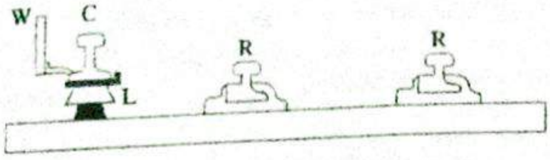


Fig. 43.6

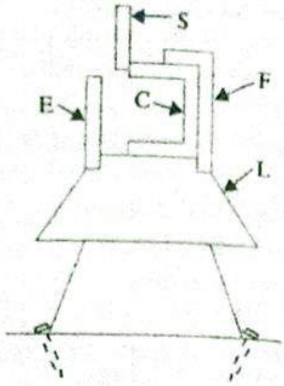


Fig. 43.7

Fig. 43.7 shows the side contact rail and the method of the mounting. The conductor rail (*C*) rests upon a wooden block recessed into the top of the procelain insulator *L*. Current is collected by steel shoes (*S*) which are kept pressed on the contact rail by springs. *E* and *F* are the guards which rest upon ledges on the insulator.

43.26. Types of Railway Services

There are three types of passenger services offered by the railways :

1. **City or Urban Service.** In this case, there are frequent stops, the distance between stops being nearly 1 km or less. Hence, high acceleration and retardation are essential to achieve moderately high schedule speed between the stations.

2. **Suburban Service.** In this case, the distance between stops averages from 3 to 5 km over a distance of 25 to 30 km from the city terminus. Here, also, high rates of acceleration and retardation are necessary.

3. **Main Line Service.** It involves operation over long routes where stops are infrequent. Here, operating speed is high and accelerating and braking periods are relatively unimportant.

On goods traffic side also, there are three types of services (*i*) main-line freight service (*ii*) local or pick-up freight service and (*iii*) shunting service.

43.27. Train Movement

The movement of trains and their energy consumption can be conveniently studied by means of speed/time and speed/distance curves. As their names indicate, former gives speed of the train at various ***times*** after the start of the run and the later gives speed at various ***distances*** from the starting point. Out of the two, speed/time curve is more important because

1. its slope gives acceleration or retardation as the case may be.
2. area between it and the horizontal (*i.e.* time) axis represents the distance travelled.
3. energy required for propulsion can be calculated if resistance to the motion of train is known.

43.28. Typical Speed/Time Curve

Typical speed/time curve for electric trains operating on passenger services is shown in Fig. 43.8. It may be divided into the following **five** parts :

1. Constant Acceleration Period (0 to t_1)

It is also called notching-up or starting period because during this period, starting resistance of the motors is gradually cut out so that the motor current (and hence, tractive effort) is maintained nearly constant which produces constant acceleration alternatively called 'rheostatic acceleration' or 'acceleration while notching'.

2. Acceleration on Speed Curve (t_1 to t_2)

This acceleration commences after the starting resistance has been all cut-out at point t_1 and full supply voltage has been applied to the motors. During this period, the motor current and torque decrease as train speed increases. Hence, acceleration gradually ***decreases*** till torque developed by motors exactly balances that due to resistance to the train motion. The shape of the portion *AB* of the speed/time curve depends primarily on the torque/speed characteristics of the traction motors.

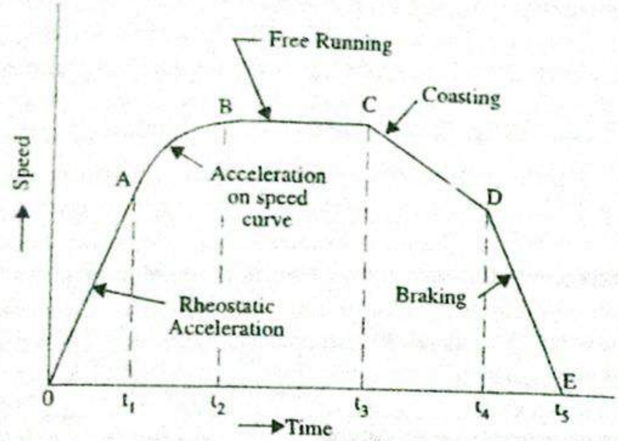


Fig. 43.8

3. Free-running Period (t_2 to t_3)

The train continues to run at the speed reached at point t_2. It is represented by portion *BC* in Fig. 43.8 and is a constant-speed period which occurs on level tracks.

4. Coasting (t_3 to t_4)

Power to the motors is cut off at point t_3 so that the train runs under its momentum, the speed gradually falling due to friction, windage etc. (portion *CD*). During this period, retardation remains practically constant. Coasting is desirable because it utilizes some of the kinetic energy of the train which would, otherwise, be wasted during braking. Hence, it helps to reduce the energy consumption of the train.

5. Braking (t_4 to t_5)

At point t_4, brakes are applied and the train is brought to rest at point t_5.

It may be noted that coasting and braking are governed by train resistance and allowable retardation respectively.

43.29. Speed/Time Curves for Different Services

Fig. 43.9 (*a*) is representative of city service where relative values of acceleration and retardation are high in order to achieve moderately high average speed between stops. Due to short distances between stops, there is no possibility of free-running period though a short coasting period is included to save on energy consumption.

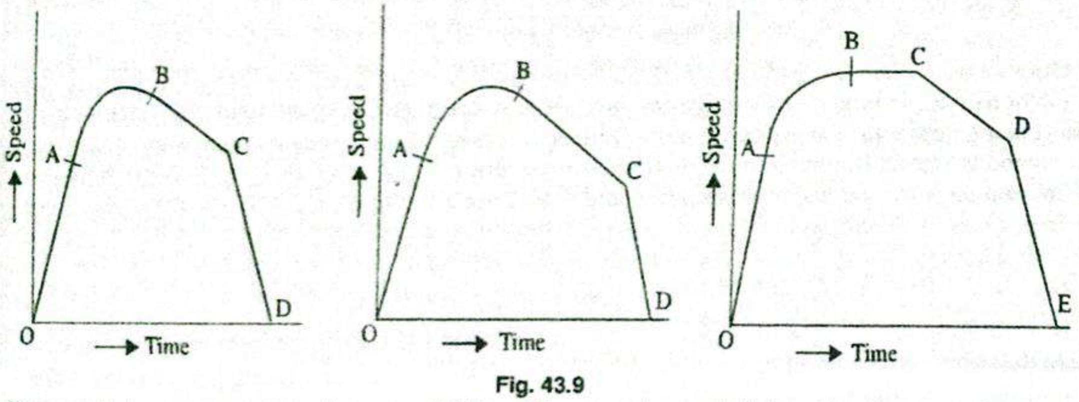


Fig. 43.9

In suburban services [Fig. 43.9 (*b*)], again there is no free-running period but there is comparatively ***longer*** coasting period because of longer distances between stops. In this case also, relatively high values of acceleration and retardation are required in order to make the service as attractive as possible.

For main-line service [Fig. 43.9 (c)], there are long periods of free-running at high speeds. The accelerating and retardation periods are relatively unimportant.

43.30. Simplified Speed/Time Curve

For the purpose of comparative performance for a given service, the actual speed/time curve of Fig. 43.8 is replaced by a simplified speed/time curve which does not involve the knowledge of motor characteristics. Such a curve has simple geometric shape so that simple mathematics can be used to find the relation between acceleration, retradation, average speed and distance etc. The simple curve would be fairly accurate provided it (*i*) ***retains the same acceleration and retardation and*** (*ii*) ***has the same area as the actual speed/time curve***. The simplified speed/time curve can have either of the two shapes :

(*i*) trapezoidal shape OA_1B_1C of Fig. 43.10 where speed-curve running and coasting periods of the actual speed/time curve have been replaced by a constant-speed period.

(*ii*) quadrilateral shape OA_2B_2C where the same two periods are replaced by the extensions of initial constnat acceleration and coasting periods.

It is found that trapezodial diagram OA_1B_1C gives simpler relationships between the principal quantities involved in train movement and also gives closer approximation of actual energy consumed during ***main-line service on level track***. On the other hand, quadrilateral diagram approximates more closely to the actual conditions in ***city and suburban services***.

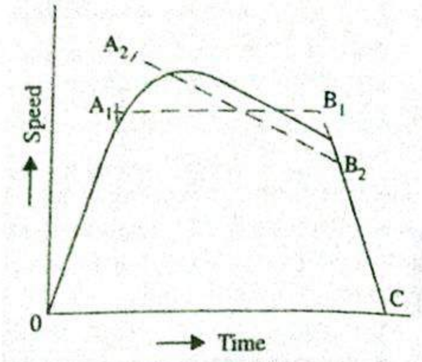


Fig. 43.10

43.31. Average and Schedule Speed

While considering train movement, the following three speeds are of importance :

1. **Crest Speed.** It is the maximum speed (V_m) attained by a train during the run.

2. **Average Speed** $= \dfrac{\text{distance between stops}}{\text{actual time of run}}$

In this case, only running time is considered but *not the stop time*.

3. **Schedule Speed** $= \dfrac{\text{distance between stops}}{\text{actual time of run + stop time}}$

Obviously, schedule speed can be obtained from average speed by including the duration of stops. For a given distance between stations, higher values of acceleration and retardation will mean lesser running time and, consequently, higher schedule speed. Similarly, for a given distance between stations and for fixed values of acceleration and retardation, higher crest speed will result in higher schedule speed. For the same value of average speed, increase in duration of stops decreases the schedule speed.

43.32. SI Units in Traction Mechanics

In describing various quantities involved in the mechanics of train movement, only the latest SI system will be used. Since SI system is an 'absolute system', only absolute units will be used while gravitational units (used hitherto) will be discarded.

1.	**Force.**	It is measured in newton (N)
2.	**Mass.**	Its unit is kilogram (kg). Commonly used bigger units is tonne (*t*). 1 tonne = 1000 kg

3. **Energy.** Its basic unit is joule (J). Other units often employed are watt-hour (Wh) and kilowatt-hour (kWh).

$$1\ \text{Wh} = 1\frac{\text{J}}{\text{s}} \times 3600\ \text{s} = 3600\ \text{J} = 3.6\ \text{kJ}$$

$$1\text{kWh} = 1000 \times 1\frac{\text{J}}{\text{s}} \times 3600\ \text{s} = 36 \times 10^5\ \text{J} = 3.6\ \text{MJ}$$

4. **Work.** Its unit is the same as that of energy.

5. **Power.** Its unit is watt (W) which equals 1 J/s. Other units are kilowatt (kW) and megawatt (MW).

6. **Distance.** Its unit is metre. Other unit often used is kilometre (km).

7. **Velocity.** Its absolute unit is metre per second (m/s). If velocity is given in km/h (or km.ph), it can be easily converted into the SI unit of m/s by multiplying it with a factor of (1000/3600) = 5/18 = 0.2778. For example, 72 km.ph = 72 × 5/18 = 72 × 0.2778 = 20 m/s.

8. **Acceleration.** Its unit is metre/second^2 (m/s^2). If acceleration is given in km/h/s (or km-ph.ps), then it can be converted into m/s^2 by simply multiplying it by the factor (1000/3600) = 5/18 = 0.2778 *i.e.* the same factor as for velocity.

For example, 1.8 km.ph.ps = 1.8 × 5/18 = 1.8 × 0.2778 = 0.5 m/s^2

43.33. Confusion Regarding Weight and Mass of a Train

Many students often get confused regarding the correct meaning of the terms 'weight' and 'mass' and their units while solving numericals on train movement particularly when they are not expressed clearly and consistently in their absolute units. It is primarily due to the mixing up of absolute units with gravitational units. There would be no confusion at all if ***we are consistent in using only absolute units*** as required by the SI system of units which disallows the use of gravitational units.

Though this topic was briefly discussed earlier, it is worth repeating here.

1. **Mass (M).** It is the quantity of matter contained in a body.

   Its absolute unit is kilogram (kg). Other multiple in common use is tonne.

2. **Weight (W).** It is the force with which earth pulls a body downwards.

The weight of a body can be expressed in (*i*) the ***absolute*** unit of newton (N) or (*ii*) the ***gravitational*** unit of kilogram-weight (kg. wt) which is often writing as 'kgf' in engineering literature.

Another still bigger ***gravitational*** unit commonly used in traction work is tonne-weight (t-wt)

$$1\ \text{t-wt} = 1000\ \text{kg-wt} = 1000 \times 9.8\ \text{N} = 9800\ \text{N}$$

(*i*) Absolute Unit of Weight

It is called newton (N) whose definition may be obtained from Newton's Second Law of Motion.

Commonly used multiple is kilo-newton (kN). Obviously, 1 kN = 1000 N = 10^3 N.

For example, if a mass of 200 kg has to be given an acceleration of 2.5 m/s^2, force required is $F = 200 \times 2.5 = 500$ N.

If a train of mass 500 tonne has to be given an acceleration of 0.6 m/s^2, force required is

$$F = ma = (500 \times 1000) \times 0.6 = 300{,}000\ \text{N} = 300\ \text{kN}$$

(*ii*) Gravitational Unit of Weight

It is 'g' times bigger than newton. It is called kilogram-weight (kg.wt.)

$$1\ \text{kg.wt} = g\ \text{newton} = 9.81\ \text{N} \cong 9.8\ \text{N}$$

Unfortunately, the word 'wt' is usually omitted from kg-wt when expressing the weight of the body on the assumption that it can be understood or inferred from the language used.

Take the statement "a body has a *weight* of 100 kg". It looks as if the weight of the body has been expressed in terms of the mass unit 'kg'. To avoid this confusion, statement should be 'a body has a weight of 100 kg. wt.' But the first statement is justified by the writers on the ground that since the word 'weight' has already been used in the statement, it should be automatically understood by the readers that 'kg' is not the 'kg' of mass but is kg-wt. It would be mass kg if the statement is 'a body has a mass of 100 kg'. Often kg-wt is written as 'kgf' where 'f' is the first letter of the word force and is added to distinguish it from kg of mass.

Now, consider the statement "a body *weighing* 500 kg travels with a speed of 36 km/h........"

Now, weight of the body W = 500 kg wt. = 500 × 9.8 N

Since we know the weight of the body, we can find its mass from the relation $W = mg$. But while using this equation, it is essential that we must ***consistently*** use the ***absolute*** units only. In this equation, W must be in newton (not in kg. wt), m in kg and g in m/s^2.

$\therefore \quad 500 \times 9.8 = m \times 9.8 \; ; \qquad \therefore \quad m = 500$ kg

It means that a body which *weighs* 500 kg (wt) has a *mass* of 500 kg.

As a practical rule, weight of a body in **gravitational** units is numerically equal to its mass in **absolute** units. This simple fact must be clearly understood to avoid any confusion between weight and mass of a body.

A train which weighs 500 tonne has a mass of 500 tonne as proved below :

train weight, $\quad W = 500$ tonne-wt $= 500 \times 1000$ kg-wt $= 500 \times 1000 \times 9.8$ N

Now, $\quad W = mg \; ; \qquad \therefore \quad 500 \times 1000 \times 9.8 = m \times 9.8$

$\therefore \quad m = 500 \times 1000$ kg $= 500 \times 1000/1000 = 500$ tonne

To avoid this unfortunate confusion, it would be helpful to change our terminology. For example, instead of saying "a train weighing 500 tonne is........" it is better to say "a 500-t train is ...." or "a train having a mass of 500 t is ...."

In order to remove this confusion, SI system of units has disallowed the use of gravitational units. There will be no confusion if ***we consistently use only absolute units.***

43.34. Quantities Involved in Traction Mechanics

Following principal quantities are involved in train movement :

D = distance between stops

M_e = effective mass of the train

W_e = effective weight of the train

β_c = retardation during coasting

V_a = average speed

t = total time for the run

t_2 = time of free running $= t - (t_1 + t_3)$

F_t = tractive effort

M = dead mass of the train

W = dead weight of the train

α = accelaration during starting period

β = retardation during braking

V_m = maximum (or crest) speed.

t_1 = time of acceleration

t_3 = time of braking

T = torque

43.35. Relationship Between Principal Quantities in Trapezoidal Diagram

As seen from Fig. 43.11.

$$\alpha = V_m / t_1 \qquad \text{or} \qquad t_1 = V_m / \alpha$$

$$\beta = V_m / t_3 \qquad \text{or} \qquad t_3 = V_m / \beta$$

As we know, total distance D between the two stops is given by the area of trapezium $OABC$.

$$\begin{aligned} \therefore \quad D &= \text{area } OABC \\ &= \text{area } OAD + \text{area } ABED + \text{area } BCE \\ &= \frac{1}{2} V_m t_1 + V_m t_2 + \frac{1}{2} V_m t_3 \end{aligned}$$

$$= \frac{1}{2} V_m t_1 + V_m[t-(t_1+t_3)] + \frac{1}{2} V_m t_3$$

$$= V_m \left[\frac{t_1}{2} + t - t_1 - t_3 + \frac{t_3}{2}\right]$$

$$= V_m \left[t - \frac{1}{2}(t_1+t_3)\right]$$

$$= V_m \left[t - \frac{V_m}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right]$$

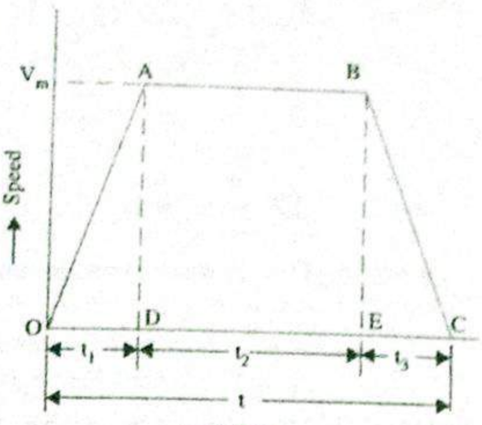


Fig. 43.11

Let, $K = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$. Substituting this value of K in the above equation, we get

$$D = V_m(t - KV_m)$$

or $KV_m^2 - V_m t + D = 0$ ...(*i*)

$$\therefore \quad V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

Rejecting the positive sign which gives impracticable value, we get

$$V_m = \frac{t \pm \sqrt{t^2 - 4KD}}{2K}$$

From Eq. (*i*) above, we get

$$KV_m^2 = V_m t - D \quad \text{or} \quad K = \frac{t}{V_m} - \frac{D}{V_m^2} = \frac{D}{V_m^2}\left(V_m \cdot \frac{t}{D} - 1\right)$$

Now, $V_a = \frac{D}{t}$. $\therefore \quad K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right)$

Obviously, if V_m, V_a and D are given, then value of K and hence of α and β can be found (Ex. 43.2).

43.36. Relationship Between Principal Quantities in Quadrilateral Diagram

The diagram is shown in Fig. 43.12. Let β_c represent the retardation during coasting period. As before,

$$t_1 = V_1/\alpha,\ t_2 = (V_2 - V_1)/\beta_c \text{ and } t_3 = V_2/\beta$$

$$D = \text{area } OABC$$

$$= \text{area } OAD + \text{area } ABED + \text{area } BCE$$

$$= \frac{1}{2}V_1 t_1 + t_2\left(\frac{V_1+V_2}{2}\right) + \frac{1}{2}V_2 t_3$$

$$= \frac{1}{2}V_1(t_1+t_2) + \frac{1}{2}V_2(t_2+t_3)$$

$$= \frac{1}{2}V_1(t-t_3) + \frac{1}{2}V_2(t-t_1)$$

$$= \frac{1}{2}t(V_1+V_2) - \frac{V_1 t_1}{2} - \frac{V_1 t_3}{2}$$

$$= \frac{1}{2}t(V_1+V_2) - \frac{1}{2}V_1 V_2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)$$

$$= \frac{1}{2}t(V_1+V_2) - KV_1V_2$$

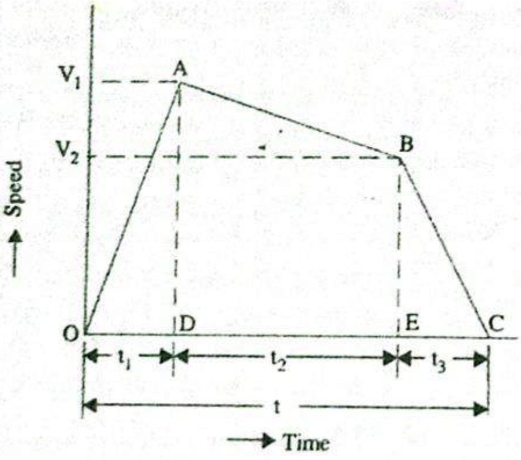


Fig. 43.12

where $K = \frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = \frac{\alpha+\beta}{2\alpha\beta}$ Also, $\beta_c = \frac{(V_1 - V_2)}{t_2}$

$$\therefore \quad V_2 = V_1 - \beta_c t_2 = V_1 - \beta_c (t - t_1 - t_3)$$

$$= V_1 - \beta_c \left(t - \frac{V_1}{\alpha} - \frac{V_2}{\beta}\right) = V_1 \beta_c \left(t - \frac{V_1}{\alpha}\right) + \beta_c \frac{V_2}{\beta}$$

or $V_2\left(1 - \frac{\beta_c}{\beta}\right) = V_1 - \beta_c\left(t - \frac{V_1}{\alpha}\right)$ $\quad \therefore \quad V_2 = \frac{V_1 - \beta_c (t - V_1/\alpha)}{(1 - \beta_c/\beta)}$

Example 43.1. *A suburban train runs with an average speed of 36 km/h between two stations 2 km apart. Values of acceleration and retardation are 1.8 km/h/s and 3.6 km/h/s.*

Compute the maximum speed of the train assuming trapezoidal speed/time curve.

(Electric Traction, Punjab Univ. 1994)

Solution. Now, $V_a = 36$ km/h $= 36 \times 5/18 = 10$ m/s

$\alpha = 1.8$ km/h/s $= 1.8 \times 5/18 = 0.5$ m/s^2, $\beta = 3.6$ km/h/s $= 3.6 \times 5/18 = 1.0$ m/s^2

$t = D/V_a = 2000/10 = 200$ s ; $K = (\alpha + \beta)/2\alpha\beta = (0.5 + 1.0)/2 \times 0.5 \times 1 = 1.5$

$$\therefore \quad V_m = \frac{t - \sqrt{t^2 - 4KD}}{2K} = \frac{200 - \sqrt{200^2 - 4 \times 1.5 \times 2000}}{2 \times 1.5}$$

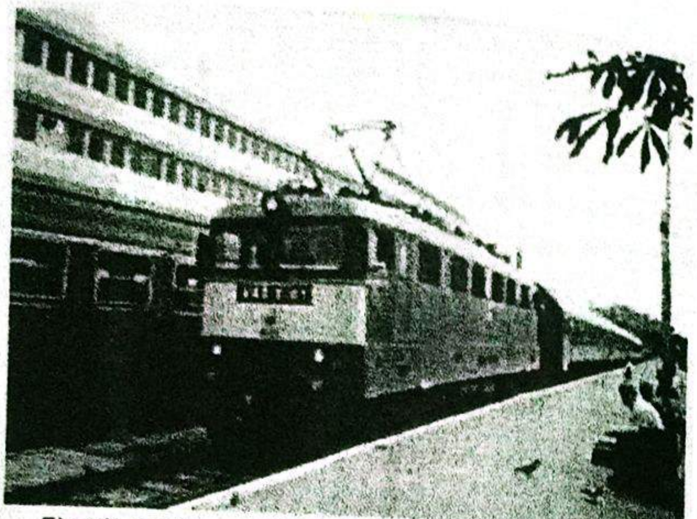
$= 11$ m/s $= 11 \times 18/5$

$=$ **39.6 km/h**

Example 43.2. *A train is required to run between two stations 1.5 km apart at a schedule speed of 36 km/h, the duration of stops being 25 seconds. The braking retardation is 3 km/h/s. Assuming a trapezoidal speed/time curve, calculate the acceleration if the ratio of maximum speed to average speed is to be 1.25*

(Elect. Power, Bombay Univ. 1980)

Solution. Here, $D = 1500$ m ; schedule speed $= 36$ km/h $= 36 \times 5/18 = 10$ m/s



Electric traction provides high starting torque and low maintenance costs making it the best choice for trains

$\beta' = 3$ km/h/s $= 3 \times 5/18 = 5/6$ m/s^2

Schedule time of run $= 1500/10 = 150$ s ; Actual time of run $= 150 - 25 = 125$ s

$\therefore \quad V_a = 1500/125 = 12$ m/s ; $V_m = 1.25 \times 12 = 15$ m/s

$$\text{Now,} \quad K = \frac{D}{V_m^2}\left(\frac{V_m}{V_a} - 1\right) = \frac{1500}{15^2}(1.25 - 1) = \frac{5}{3}$$

$$\text{Also,} \quad K = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) \quad \text{or} \quad \frac{5}{3} = \frac{1}{2}\left(\frac{1}{\alpha} + \frac{6}{5}\right)$$

$\therefore \quad \alpha = 0.47$ m/s^2 $= 0.47 \times 18/5 =$ **1.7 km/h/s**

Example 43.3. *Find the schedule speed of an electric train for a run of 1.5 km if the ratio of its maximum to average speed is 1.25. It has a braking retardation of 3.6 km/h/s, acceleration of 1.8 km/h/s and stop time of 21 second. Assume trapezoidal speed/time curve.*

Solution. $\alpha = 1.8 \times 5/18 = 0.5\ \text{m/s}^2,\ \beta = 3.6 \times 5/18 = 1.0\ \text{m/s}^2$

$D = 1.5\ \text{km} = 1500\ \text{m}$

$$K = \frac{1}{2}\left(\frac{1}{0.5} + \frac{1}{1}\right) = \frac{3}{2} \qquad \text{Now,} \quad K = \frac{D}{V_{m^2}}\left(\frac{V_m}{V_a} - 1\right)$$

or
$$V_m^2 = \frac{D}{K}\left(\frac{V_m}{V_a} - 1\right) \qquad \therefore \quad V_m^2 = \frac{1500}{3/2}(1.25 - 1) = 250\ ;\ V_m = 15.8\ \text{m/s}$$

$V_a = V_m/1.25 = 15.8/1.25 = 12.6$ m/s

Actual time of run = 1500/12.6 = 119 seconds

Schedule time = 119 + 21 = 140 second

∴ Schedule speed = 1500/140 = 10.7 m/s = **38.5 km/h**

Example 43.4. *A train runs between two stations 1.6 km apart at an average speed of 36 km/h. If the maximum speed is to be limited to 72 km/h, acceleration to 2.7 km/h/s, coasting retardation to 0.18 km/h/s and braking retardation to 3.2 km/h/s, compute the duration of acceleration, coasting and braking periods.*

Assume a simplified speed/time curve.

Solution. Given : $D = 1.6\ \text{km} = 1600\ \text{m}$, $V_a = 36\ \text{km/h} = 10\ \text{m/s}$

$V_1 = 72\ \text{km/h} = 20\ \text{m/s}$; $\alpha = 2.7\ \text{km/h/s} = 0.75\ \text{m/s}^2$

$\beta_c = 0.18\ \text{km/h/s} = 0.05\ \text{m/s}^2$; $\beta = 3.6\ \text{km/h/s} = 1.0\ \text{m/s}^2$

With reference to Fig. 43.12, we have

Duration of acceleration, $t_1 = V_1/\alpha = 20/0.75 = 27$ s

Actual time of run, t = 1600/10 = **160 s**

Duration of braking, $t_3 = V_2/1.0 = V_2$ second

Duration of coasting, $t_2 = (V_1 - V_2)/\beta_c = (20 - V_2)/0.05 = (400 - 20\ V_2)$ second

Now, $t = t_1 + t_2 + t_3$ or $160 = 27 + (400 - 20\ V_2) + V_2$ $\therefore\ V_2 = 14$ m/s

∴ $t_2 = (20 - 14)/0.05 =$ **120 s** ; $t_3 = 14/1.0 =$ **14 s**

43.37. Tractive Effort for Propulsion of a Train

The tractive effort (F_t) is the force developed by the traction unit at the rim of the driving wheels for moving the unit itself and its train (trailing load). The tractive effort required for train propulsion on a *level track* is

$$F_t = F_a + F_r$$

If gradients are involved, the above expression becomes

$$F_t = F_a + F_g + F_r \qquad \text{— for ascending gradient}$$
$$= F_a - F_g + F_r \qquad \text{— for descending gradient}$$

where
F_a = force required for giving linear acceleration to the train
F_g = force required to overcome the effect of gravity
F_r = force required to overcome resistance to train motion.

(*a*) Value of F_a

If M is the dead (or stationary) mass of the train and a its linear acceleration, then

$$F_a = Ma$$

Since a train has rotating parts like wheels, axles, motor armatures and gearing etc., its *effective* (or accelerating) mass M_e is more (about 8 – 15%) than its stationary mass. These parts have to be given angular acceleration at the same time as the whole train is accelerated in the linear direction. Hence, $F_e = M_e a$

(*i*) If M_e is in kg and α in m/s^2, then $F_a = M_e\, a$ newton

(*ii*) If M_e is in tonne and α in km/h/s, then converting them into absolute units, we have

$$F_a = (1000\, M_e) \times (1000/3600)\, a = 277.8\, M_e\, a \text{ newton}$$

(*b*) **Value of F_g**

As seen from Fig. 43.13, $F_g = W \sin\theta = Mg \sin\theta$

In railway practice, gradient is expressed as the rise (in metres) a track distance of 100 m and is called percentage gradient.

$$\therefore \quad \%\, G = \frac{BC}{AC/100} = 100\,\frac{BC}{AC} = 100 \sin\theta$$

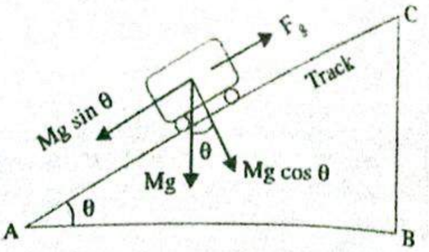


Fig. 43.13

Substituting the value of sin θ in the above equation, we get

$$F_g = Mg\, G/100 = 9.8 \times 10^{-2}\, MG$$

(*i*) When M is in kg, $F_g = 9.8 \times 10^{-2}\, MG$ newton

(*ii*) When M is given in tonne, then

$$F_g = 9.8 \times 10^{-2}\,(1000\, M)\, G = 98\, MG \text{ newton}$$

(*c*) **Value of F_r**

Train resistance comprises all those forces which oppose its motion. It consists of mechanical resistance and wind resistance. Mechanical resistance itself is made up of internal and external resistances. The internal resistance comprises friction at journals, axles, guides and buffers etc. The external resistance consists of friction between wheels and rails and flange friction etc. Mechanical resistance is almost independent of train speed but depends on its weight. The wind friction varies directly as the square of the train speed.

If r is specific resistance of the train *i.e.* resistance offered per unit mass of the train, then $F_r = M.r$.

(*i*) If r is in newton per kg of train mass and M is the train mass in kg, then

$$F_r = M.r \text{ newton}$$

(*ii*) If r is in newton per tonne train mass (N/t) and M is in tonne (t), then

$$F_r = M \text{ tonne} \times r = M_r \text{ newton*}$$

Hence, expression for total tractive effort becomes

$$F_t = F_a \pm F_g + F_r = (277.8\, \alpha\, M_e \pm 98\, MG + Mr) \text{ newton}$$

Please remember that here M is in tonne, α in km/h/s, G is in metres per 100 m of track length (*i.e.* % *G*) and r is in newton/tonne (*N/t*) of train mass.

The positive sign for F_g is taken when motion is along an ascending gradient and negative sign when motion is along a descending gradient.

43.38. Power Output from Driving Axles

If F_t is the tractive effort and v is the train velocity, then

$$\text{output power} = F_t \times \text{v}$$

(*i*) If F_t is in newton and v in m/s, then

$$\text{output power} = F_t \times \text{v watt}$$

(*ii*) If F_t is in newton and v is in km/h, then converting v into m/s, we have

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* If r is in kg (wt) per tonne train mass and M is in tonne, then $F_r = M$ tonne × (r × 9.8) newton/tonne = 9.8 Mr newton.

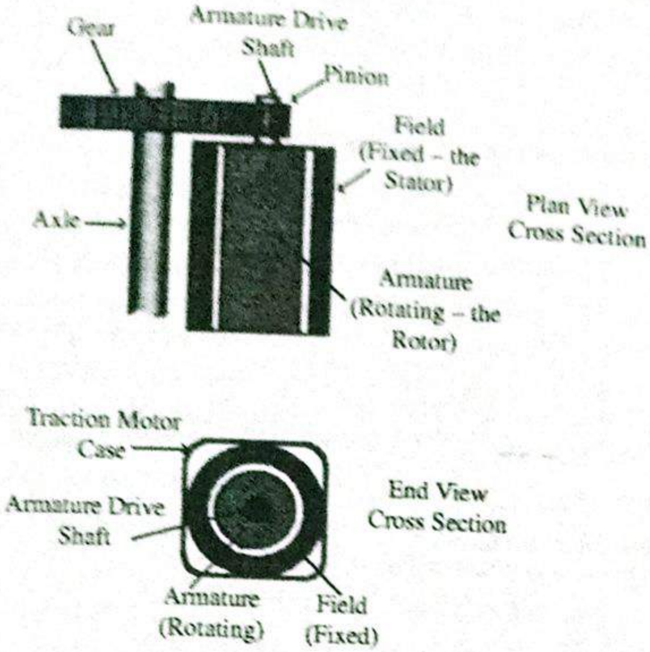

Diagram Shows how a DC motor drives the axle through a pinion and gearwheel

Traction motor schematic diagram

$$\text{output power} = F_t \times \left(\frac{1000}{3600}\right) v \text{ watt} = \frac{F_t\, v}{3600} \text{ kW}$$

If η is the efficiency of transmission gear, then power output of motors is

$$= F_t \,.\, v/\eta \text{ watt} \qquad \text{— } v \text{ in m/s}$$

$$= \frac{F_t\, v}{3600\,\eta} \text{ kW} \qquad \text{— } v \text{ in km/h}$$

43.39. Energy Output from Driving Axles

Energy (like work) is given by the product of power and time.

$$E = (F_t \times v) \times t = F_t \times (v \times t) = F_t \times D$$

where D is the distance travelled in the direction of tractive effort.

Total energy output from driving axles for the run is

$$E = \text{energy during acceleration} + \text{energy during free run}$$

As seen from Fig. 43.11

$$E = F_t \times \text{area } OAD + F_t' \times \text{area } ABED = F_t \times \frac{1}{2}\, V_m t_1 + F_t' \times \frac{1}{2}\, V_m t_2$$

where F_t is the tractive effort during accelerating period and F_t' that during free-running period. Incidentally, F_t will consist of all the three components given in Art. 43.37 whereas F_t' will consist of $(98\, MG + Mr)$ provided there is an ascending gradient.

43.40. Specific Energy Output

It is the energy output of the driving wheel expressed in watt-hour (Wh) per tonne-km (t-km) of the train. It can be found by first converting the energy output into Wh and then dividing it by the mass of the train in tonne and route distance in km.

Hence, unit of specific energy output generally used in railway work is : Wh/tonne-km (Wh/t-km).

43.41. Evaluation of Specific Energy Output

We will first calculate the total energy output of the driving axles and then divide it by train mass in tonne and route length in km to find the specific energy output. It will be presumed that :

(*i*) there is a gradient of G throughout the run and

(*ii*) power remains ON upto the end of free run in the case of trapezoidal curve (Fig. 43.11) and upto the accelerating period in the case of quadrilateral curve (Fig. 43.12).

Now, output of the driving axles is used for the following purposes :

1. for accelerating the train
2. for overcoming the gradient
3. for overcoming train resistance.

(*a*) **Energy required for train acceleration (E_a)**

As seen from trapezoidal diagram of Fig. 43.11.

$$E_a = F_a \times \text{distance } OAD = 277.8\,\alpha\, M_e \times \frac{1}{2}\, V_m . t_1 \text{ joules}$$

$$= 277.8\,\alpha\, M_e \times \frac{1}{2}\, V_m \times \frac{V_m}{\alpha} \text{ joules} \qquad \left(\text{फ्ट}\; t_1 = \frac{V_m}{\alpha}\right)$$

$$= 277.8\,\alpha\, M_e \times \left[\frac{1}{2}.\frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{ joules}$$

It will be seen that since V_m is in km/h, it has been converted into m/s by multiplying it with the conversion factor of (1000/3600). In the case of (V_m /*t*), conversion factors for V_m and a being the same, they cancel out. Since 1 Wh = 3600 J.

$$\therefore \quad E_a = 277.8\,\alpha\, M_e \left[\frac{1}{2}.\frac{V_m \times 1000}{3600} \times \frac{V_m}{\alpha}\right] \text{Wh} = 0.01072\,\frac{V_m^2}{M_e}\text{ Wh}$$

(*b*) **Energy required for over coming gradient (E_g)**

$$E_g = F_g \times D'$$

where '*D*'' is the *total distance over which power remains* ON. Its maximum value equals the distance represented by the area *OABE* in Fig. 43.11 *i.e.* from the start to the end of free-running period in the case of trapezoidal curve [as per assumption (*i*) above].

Substituting the value of F_g from Art. 43.37, we get

$$E_g = 98\, MG.\,(1000\, D')\text{ joules} = 98{,}000\, MGD'\text{ joules}$$

It has been assumed that D' is in km.

When expressed in Wh, it becomes

$$E_g = 98{,}000\, MGD'\,\frac{1}{3600}\text{ Wh} = 27.25\, MGD'\text{ Wh}$$

(*c*) **Energy required for overcoming resistance (E_r)**

$$E_r = F_r \times D' = M\,.\,r \times (1000\, D')\text{ joules} \qquad \text{— } D' \text{ in km}$$

$$= \frac{1000\, Mr\, D'}{3600}\text{ Wh} = 0.2778\, Mr\, D'\text{ Wh} \qquad \text{— } D' \text{ in km}$$

∴ total energy output of the driving axles is

$$\begin{aligned} E &= E_a + E_g + E_r \\ &= (0.01072\, V_m^2/M_e + 27.25\, MGD' + 0.2778\, Mr\, D' \text{ Wh} \end{aligned}$$

Specific energy output

$$E_{spo} = \frac{E}{M \times D} \qquad \text{— } D \text{ is the } \textit{total} \text{ run length}$$

$$= \left(0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 27.25\, G \frac{D'}{D} + 0.2778\, r \frac{D'}{D}\right) \text{Wh/t-km}$$

It may be noted that *if there is no gradient*, then

$$E_{spo} = \left(0.01072 \frac{V_m^2}{D} \cdot \frac{M_e}{M} + 0.2778\, r \frac{D'}{D}\right) \text{Wh/t-km}$$

Alternative Method

As before, we will consider the trapezoidal speed/time curve. Now, we will calculate energy output not **force-wise** but **period-wise**.

(*i*) **Energy output during accelerating period**

$$\begin{aligned} E_a' &= F_t \times \text{distance travelled during accelerating period} \\ &= F_t \times \text{area } OAD \qquad \text{—Fig. 43.11} \\ &= F_t \times \frac{1}{2} V_m\, t_1 = \frac{1}{2} F_t . V_m . \frac{V_m}{\alpha} \\ &= \frac{1}{2} . F_t \left(\frac{1000}{3600} . V_m\right) . \frac{V_m}{\alpha} \text{ joules} \\ &= \frac{1}{2} . F_t \left(\frac{1000}{3600} . V_m\right) . \frac{V_m}{\alpha} . \frac{1}{3600} \text{ Wh} \end{aligned}$$

Substituting the value of F_t, we get

$$E_a = \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (277.8\, \alpha M_e + 98\, MG + Mr) \text{ Wh}$$

It must be remembered that during this period, ***all the three forces are at work*** (Art. 43.37)

(*ii*) **Energy output during free-running period**

Here, work is required only against two forces *i.e.* gravity and resistance (as mentioned earlier).

$$\begin{aligned} \text{Energy} \quad E_{fr} &= F_t' \times \text{area } ABED \qquad \text{—Fig. 43.11} \\ &= F_t' \times (V_m \times t_2) = F_t' \times \left(\frac{1000}{3600} V_m\right) . t_2 \text{ joules} \\ &= F_t' \times \left(\frac{1000}{3600} V_m\right) \times t_2 \times \frac{1}{3600} \text{ Wh} = \left(\frac{1000}{3600}\right) F_t' \times V_m\, t_2 . \frac{1}{3600} \text{ Wh} \\ &= \left(\frac{1000}{3600}\right) . F_t' \times D_{fr} \text{ Wh} = \left(\frac{1000}{3600}\right) (98\, MG + Mr)\, D_{fr} \text{ Wh} \end{aligned}$$

where D_{fr} is the distance in km travelled during the free-running period*

Total energy required is the sum of the above two energies.

$$\therefore \; E = E_a + E_{fr}$$

$$= \frac{1000}{(3600)^2} \frac{V_m^2}{2\alpha} (277.8\, \alpha . M_e + 98\, MG + Mr) + \frac{1000}{3600} (98\, MG + Mr)\, D_{fr} \text{ Wh}$$

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* D_{fr} = velocity in km/h × time in hours

= $V_m \times (t_2 / 3600)$ because times are always taken in seconds.

$$= \frac{1000}{(3600)^2} \frac{V_m^2}{2\alpha} 277.8\,\alpha M_e + \frac{1000}{(3600)^2} \cdot \frac{V_m^2}{2\alpha} (98\,MG + Mr) + \frac{1000}{3600} (98\,MG + Mr) \,.\, D_{fr}\ \text{Wh}$$

$$= 0.01072\, V_m^2 \,.\, M_e + \frac{1000}{3600} (98\,MG + Mr) \left(\frac{V_m^2}{2\alpha \times 3600} + D_{fr} \right) \text{Wh}$$

Now,

$$\frac{V_m^2}{2\alpha \times 3600} = \frac{1}{2}\left(\frac{V_m}{3600}\right) \cdot \frac{V_m}{\alpha} = \frac{1}{2}\left(\frac{V_m}{3600}\right) \cdot t_1$$

$$= \text{distance travelled during accelerating period } i.e.\ D_a$$

$\therefore$

$$E = 0.01072\, V_m^2 \,.\, M_e + \frac{1000}{3600} (98\,MG + Mr)(D_a + D_{fr})\ \text{Wh}$$

$$= 0.01072\, V_m^2 \,.\, M_e + (27.25\,MG + 0.2778\,Mr)\, D'\ \text{Wh}$$

It is the same expression as found above.

43.42. Energy Consumption

It equals the total energy input to the traction motors from the supply. It is usually expressed in Wh which equals 3600 J. It can be found by dividing the energy output of the driving wheels with the combined efficiency of transmission gear and motor.

$$\therefore \quad \text{energy consumption} = \frac{\text{output of driving axles}}{\eta_{motor} \times \eta_{gear}}$$

43.43. Specific Energy Consumption

It is the energy consumed (in Wh) per tonne mass of the train per km length of the run.

Specific energy consumption,

$$E_{spc} = \frac{\text{total energy consumed in Wh}}{\text{train mass in tonne} \times \text{run length in km}} = \frac{\text{specific energy output}}{\eta}$$

where η = overall efficiency of transmission gear and motor = $\eta_{gear} \times \eta_{motor}$

As seen from Art. 43.41, specific energy consumption is

$$E_{spc} = \left(0.01072 . \frac{V_m^2}{\eta D} . \frac{M_e}{M} + 27.25 \frac{G}{\eta} . \frac{D'}{D} + 0.2778 \frac{r}{\eta} . \frac{D'}{D} \right) \text{Wh/t-km}$$

If no gradient is involved, then specific enrgy consumption is

$$E_{spc} = \left(0.01072 . \frac{V_m^2}{\eta D} . \frac{M_e}{M} + 0.2778 \frac{r}{\eta} . \frac{D'}{D} \right) \text{Wh/t-km}$$

The specific energy consumption of a train running at a given schedule speed is influenced by **1.** Distance between stops **2.** Acceleration **3.** Retardation **4.** Maximum speed **5.** Type of train and equipment **6.** Track configuration.

43.44. Adhesive Weight

It is given by the total weight carried on the driving wheels. Its value is $W_a = x\,W$, where W is dead weight and x is a fraction varying from 0.6 to 0.8.

43.45. Coefficient of Adhesion

Adhesion between two bodies is due to interlocking of the irregularities of their surfaces in contact. The adhesive weight of a train is ***equal to the total weight to be carried on the driving*** *wheels*. It is less than the dead weight by about 20 to 40%.

If $x = \dfrac{\text{adhesive weight, } W_a}{\text{dead weight } W}$, then, $W_a = x\,W$

Let, F_t = tractive effort to slip the wheels

or

= maximum tractive effort possible without wheel slip

Coefficient of adhesion, $\mu_a = F_t/W_a$

$\therefore$ $F_t = \mu_a\,W_a = \mu_a\,xW = \mu_a\,x\,Mg$

If M is in tonne, then

$$F_t = 1000 \times 9.8\,x\,\mu_a\,M = 9800\,\mu_a\,x\,M \text{ newton}$$

It has been found that tractive effort can be increased by increasing the motor torque but only upto a certain point. Beyond this point, any increase in motor torque does not increase the tractive effort but merely causes the driving wheels to slip. It is seen from the above relation that for increasing F_t, it is not enough to increase the kW rating of the traction motors alone but the weight on the driving wheels has also to be increased.

Adhesion also plays an important role in braking. If braking effort exceeds the adhesive weight of the vehicle, skidding takes place.

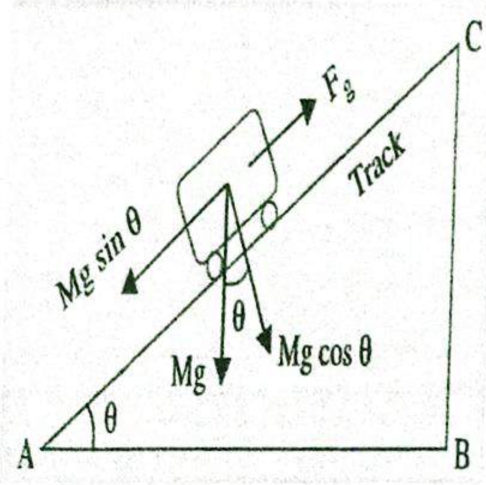

Fig. 43.14

43.46. Mechanism of Train Movement

The essentials of driving mechanism in an electric vehicle are illustrated in Fig. 43.14. The armature of the driving motor has a pinion which meshes with the gear wheel keyed to the axle of the driving wheel. In this way, motor torque is transferred to the wheel through the gear.

Let, T = torque exerted by the motor

F_1 = tractive effort at the pinion

F_t = tractive effort at the wheel

γ = gear ratio

Here, d_1, d_2 = diameters of the pinion and gear wheel respectively

D = diameter of the driving wheel

η = efficiency of power transmission from the motor to driving axle

Now, $T = F_1 \times d_1/2$ or $F_1 = 2T/d_1$

Tractive effort transferred to the driving wheel is

$$F_t = \eta F_1\left(\frac{d_2}{D}\right) = \eta \cdot \frac{2T}{d_1}\left(\frac{d_2}{D}\right) = \eta\,T\left(\frac{2}{D}\right)\left(\frac{d_2}{d_1}\right) = 2\,\gamma\eta\,\frac{T}{D}$$

For obtaining motion of the train without slipping, $F_t \le \mu_a\,W_a$ where μ_a is the coefficient of adhesion (Art. 43.45) and W_a is the adhesive weight.

Example 43.5. *The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.*

Solution. Maximum number of revolutions per second made by armature

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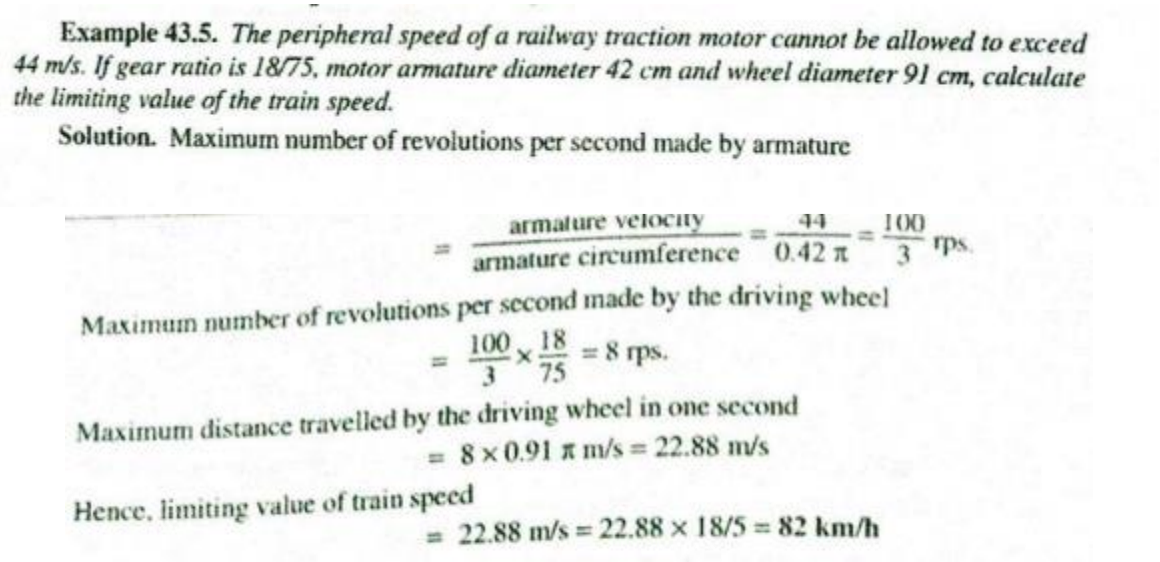
1.



Example 43.5. *The peripheral speed of a railway traction motor cannot be allowed to exceed 44 m/s. If gear ratio is 18/75, motor armature diameter 42 cm and wheel diameter 91 cm, calculate the limiting value of the train speed.*

Solution. Maximum number of revolutions per second made by armature

$$= \frac{\text{armature velocity}}{\text{armature circumference}} = \frac{44}{0.42\,\pi} = \frac{100}{3}\text{ rps.}$$

Maximum number of revolutions per second made by the driving wheel

$$= \frac{100}{3} \times \frac{18}{75} = 8\text{ rps.}$$

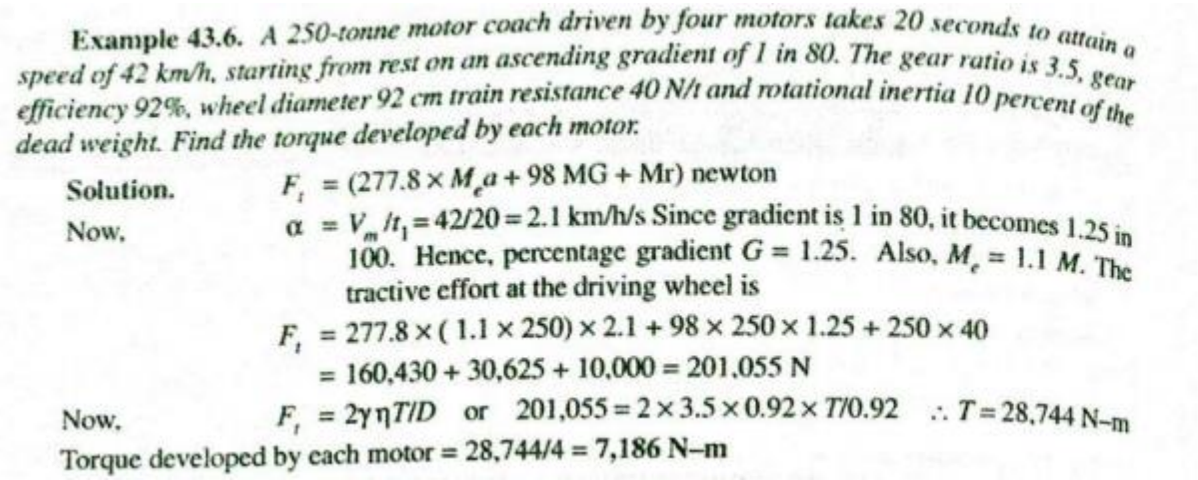
Maximum distance travelled by the driving wheel in one second

$$= 8 \times 0.91\,\pi\text{ m/s} = 22.88\text{ m/s}$$

Hence, limiting value of train speed

$$= 22.88\text{ m/s} = 22.88 \times 18/5 = \mathbf{82\ km/h}$$

2.



Example 43.6. *A 250-tonne motor coach driven by four motors takes 20 seconds to attain a speed of 42 km/h, starting from rest on an ascending gradient of 1 in 80. The gear ratio is 3.5, gear efficiency 92%, wheel diameter 92 cm train resistance 40 N/t and rotational inertia 10 percent of the dead weight. Find the torque developed by each motor.*

Solution. $F_t = (277.8 \times M_e a + 98\text{ MG} + \text{Mr})$ newton

Now, $\alpha = V_m/t_1 = 42/20 = 2.1$ km/h/s Since gradient is 1 in 80, it becomes 1.25 in 100. Hence, percentage gradient $G = 1.25$. Also, $M_e = 1.1\,M$. The tractive effort at the driving wheel is

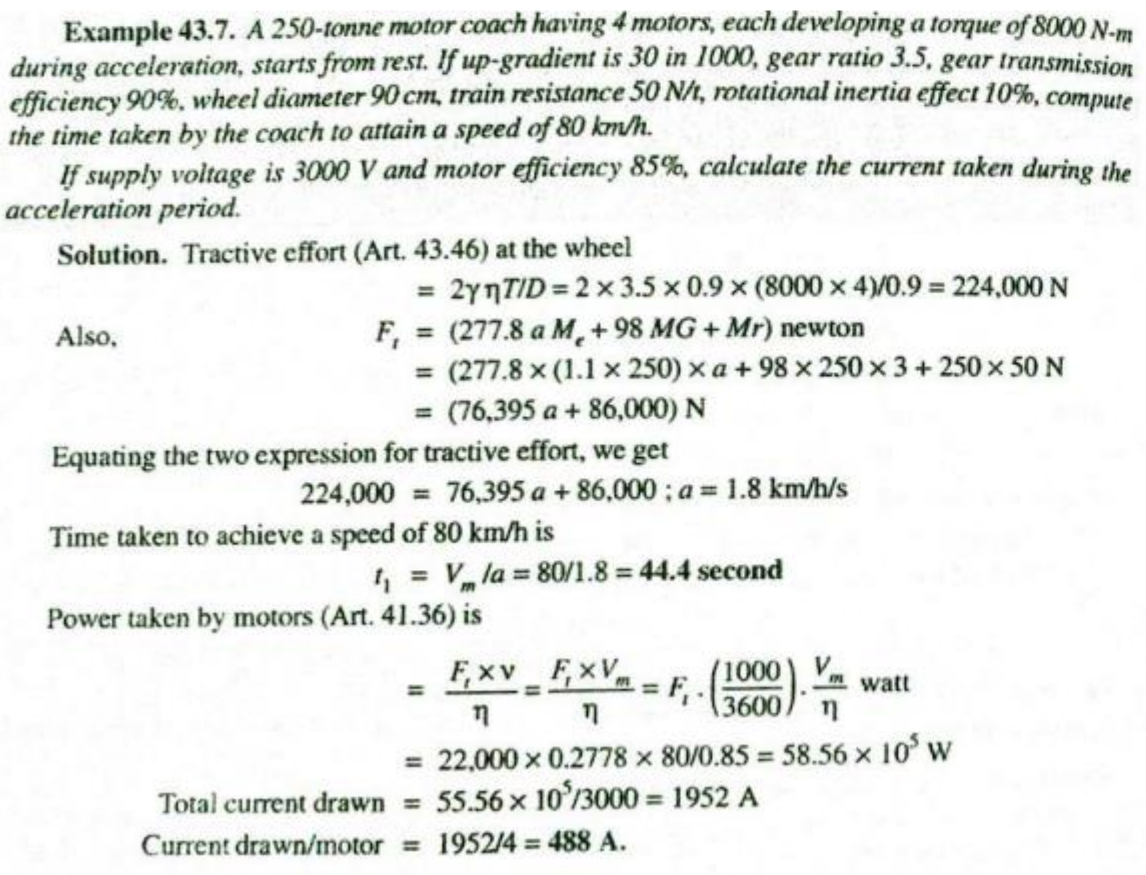
$$F_t = 277.8 \times (1.1 \times 250) \times 2.1 + 98 \times 250 \times 1.25 + 250 \times 40$$

$$= 160{,}430 + 30{,}625 + 10{,}000 = 201{,}055\text{ N}$$

Now, $F_t = 2\gamma\eta T/D$ or $201{,}055 = 2 \times 3.5 \times 0.92 \times T/0.92$ $\therefore T = 28{,}744$ N–m

Torque developed by each motor = 28,744/4 = **7,186 N–m**

3.



Example 43.7. *A 250-tonne motor coach having 4 motors, each developing a torque of 8000 N-m during acceleration, starts from rest. If up-gradient is 30 in 1000, gear ratio 3.5, gear transmission efficiency 90%, wheel diameter 90 cm, train resistance 50 N/t, rotational inertia effect 10%, compute the time taken by the coach to attain a speed of 80 km/h.*

If supply voltage is 3000 V and motor efficiency 85%, calculate the current taken during the acceleration period.

Solution. Tractive effort (Art. 43.46) at the wheel

$$= 2\gamma\eta T/D = 2 \times 3.5 \times 0.9 \times (8000 \times 4)/0.9 = 224{,}000 \text{ N}$$

Also,

$$F_t = (277.8\, a\, M_e + 98\, MG + Mr) \text{ newton}$$

$$= (277.8 \times (1.1 \times 250) \times a + 98 \times 250 \times 3 + 250 \times 50 \text{ N}$$

$$= (76{,}395\, a + 86{,}000) \text{ N}$$

Equating the two expression for tractive effort, we get

$$224{,}000 = 76{,}395\, a + 86{,}000 \; ; \; a = 1.8 \text{ km/h/s}$$

Time taken to achieve a speed of 80 km/h is

$$t_1 = V_m/a = 80/1.8 = \mathbf{44.4 \text{ second}}$$

Power taken by motors (Art. 41.36) is

$$= \frac{F_t \times v}{\eta} = \frac{F_t \times V_m}{\eta} = F_t \cdot \left(\frac{1000}{3600}\right) \cdot \frac{V_m}{\eta} \text{ watt}$$

$$= 22{,}000 \times 0.2778 \times 80/0.85 = 58.56 \times 10^5 \text{ W}$$

Total current drawn $= 55.56 \times 10^5/3000 = 1952$ A

Current drawn/motor $= 1952/4 =$ **488 A.**

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